

Formulario di trigonometria

Formula fondamentale e derivate:

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

$$\cos \alpha = \pm \frac{1}{\sqrt{1 + \tan^2 \alpha}}$$

$$\sin \alpha = \pm \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$$

Tabella angoli noti!

$\alpha(\text{gradi})$	$\alpha \text{ (rad)}$	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
0°	0	0	1	0	∞
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	∞	0

Formula di addizione:

$$\begin{aligned}\cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \\ \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \sin \beta \cos \alpha \\ \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \sin \beta \cos \alpha \\ \\ \tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\end{aligned}$$

Formula di bisezione:

$$\begin{aligned}\sin \frac{\alpha}{2} &= \pm \sqrt{\frac{1 - \cos \alpha}{2}} \\ \cos \frac{\alpha}{2} &= \pm \sqrt{\frac{1 + \cos \alpha}{2}} \\ \tan \frac{\alpha}{2} &= \begin{cases} \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \\ \frac{1 - \cos \alpha}{\sin \alpha} \\ \frac{\sin \alpha}{1 + \cos \alpha} \end{cases}\end{aligned}$$

Formula di duplicazione:

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \\ 1 - 2 \sin^2 \alpha \end{cases} \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

Formula di triplicazione:

$$\begin{aligned}\sin 3\alpha &= 3 \sin \alpha - 4 \sin^3 \alpha \\ \cos 3\alpha &= 4 \cos^3 \alpha - 3 \cos \alpha\end{aligned}$$

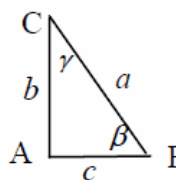
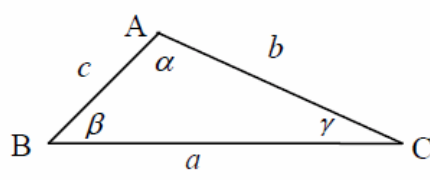
Formule parametriche:

$$\left. \begin{aligned}\sin \alpha &= \frac{2t}{1+t^2} \\ \cos \alpha &= \frac{1-t^2}{1+t^2} \\ \tan \alpha &= \frac{2t}{1-t^2}\end{aligned}\right\} \left(t = \tan \frac{\alpha}{2} \right)$$

Formule di prostaferesi:

$$\begin{aligned}\sin p + \sin q &= 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2} \\ \sin p - \sin q &= 2 \sin \frac{p-q}{2} \cos \frac{p+q}{2} \\ \cos p + \cos q &= 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2} \\ \cos p - \cos q &= -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}\end{aligned}$$

Formule per triangoli:

Triangoli rettangoli	Triangoli qualunque
	
$b = a \sin \beta = a \cos \gamma = c \tan \beta$ $a = \frac{b}{\sin \beta} = \frac{b}{\cos \gamma}; \tan \beta = \frac{b}{c}$	$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$
$c = a \sin \gamma = a \cos \beta = b \tan \gamma$ $a = \frac{c}{\sin \gamma} = \frac{c}{\cos \beta}; \tan \gamma = \frac{c}{b}$	$a^2 = b^2 + c^2 - 2bc \cos \alpha$ $b^2 = a^2 + c^2 - 2ac \cos \beta$ $c^2 = a^2 + b^2 - 2ab \cos \gamma$
	$\begin{aligned}Area(ABC) &= \frac{1}{2} bc \sin \alpha \\ &= \frac{1}{2} ac \sin \beta \\ &= \frac{1}{2} ab \sin \gamma\end{aligned}$