

ESERCIZIO N. 1.

Si calcolino i seguenti limiti:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow 0} \frac{|3x-1| - |3x+1|}{x}; & \text{b)} \quad & \lim_{x \rightarrow 0} \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{x^2}; \\ \text{c)} \quad & \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right); & \text{d)} \quad & \lim_{x \rightarrow 1} \frac{x^2-1}{x^2-2x+1}; \\ \text{e)} \quad & \lim_{x \rightarrow 8} \frac{x^{\frac{2}{3}} - 4}{x^{\frac{1}{3}} - 2}; & \text{f)} \quad & \lim_{x \rightarrow +\infty} \frac{x\sqrt{x+1}(1-\sqrt{2x+3})}{7-6x+4x^2}. \end{aligned}$$

ESERCIZIO N. 2.

Si calcolino i seguenti limiti:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow +\infty} \sqrt{x^2+x} - x; & \text{b)} \quad & \lim_{x \rightarrow 2} \frac{\sin^2(\pi x)}{x^2-4x+4}; \\ \text{c)} \quad & \lim_{x \rightarrow +\infty} x(\sqrt{x^\alpha+1} - \sqrt{x^\alpha-1}); \quad \alpha \in \mathbb{R}, \alpha \geq 0; & \text{d)} \quad & \lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{1}{2x}} \\ \text{e)} \quad & \lim_{x \rightarrow +\infty} \left(\log \left(\frac{x^2}{x+1} \right) \right)^{\frac{1}{x}}; & \text{f)} \quad & \lim_{x \rightarrow 0} \frac{2^x - 3^x}{x}; \\ \text{g)} \quad & \lim_{x \rightarrow -\infty} x(\sqrt[4]{x^4+x^2} - \sqrt[4]{x^4-x^2}); & \text{h)} \quad & \lim_{x \rightarrow 0} \frac{\log(3^x+1) - \log 2}{x}. \end{aligned}$$

ESERCIZIO N. 3.

Si calcolino i seguenti limiti:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow 0} \frac{\sqrt{1-\cos x} - x^2}{\sqrt{1-\cos x} + x^2}; & \text{b)} \quad & \lim_{x \rightarrow 0} (\sin x + \cos x)^{\frac{1}{x}}; \\ \text{c)} \quad & \lim_{x \rightarrow +\infty} \frac{\sin(\sqrt{x+1} - \sqrt{x})}{\sqrt{\text{tg}(\frac{1}{x})}}; & \text{d)} \quad & \lim_{x \rightarrow 0} \frac{\log(e + \text{tg}^2 x) - e^{x^2}}{x^2}; \\ \text{e)} \quad & \lim_{x \rightarrow +\infty} x^2(\log(x^2+x+1) - \log(x^2+x-1)); & \text{f)} \quad & \lim_{x \rightarrow +\infty} [\cos(x^4 e^{-x})]^x; \\ \text{g)} \quad & \lim_{x \rightarrow 0} f(x) \quad \text{se è noto che } |f(x)| \leq \sin^2 x \text{ per ogni } x \in]-1, 1[. \end{aligned}$$

SOLUZIONI

Es. 1:

$$\text{a)} -6; \quad \text{b)} \frac{\sqrt{2}}{2}; \quad \text{c)} \frac{1}{4};$$

$$\text{d)} \infty; \quad \text{e)} 4; \quad \text{f)} -\frac{\sqrt{2}}{4}.$$

Es. 2:

$$\text{a)} \frac{1}{2}; \quad \text{b)} \pi^2; \quad \text{c)} 0 \text{ se } \alpha > 2, 1 \text{ se } \alpha = 2, +\infty \text{ se } 0 \leq \alpha < 2;$$

$$\text{d)} \sqrt[4]{e}; \quad \text{e)} 1; \quad \text{f)} \log \frac{2}{3};$$

$$\text{g)} -\frac{1}{2}; \quad \text{h)} \frac{\log 3}{2}.$$

Es. 3:

$$\text{a)} 1; \quad \text{b)} e \text{ (si osservi che } \sin x + \cos x = \sqrt{1+2\sin x \cos x}); \quad \text{c)} \frac{1}{2};$$

$$\text{d)} \frac{1}{e} - 1; \quad \text{e)} 2; \quad \text{f)} 1; \quad \text{g)} 0.$$